Tacit Collusion in Oligopoly*

Edward J. Green,† Robert C. Marshall,‡ and Leslie M. Marx§

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Abstract

We examine the economics literature on tacit collusion in oligopoly markets and take steps toward clarifying the relation between economists’ analysis of tacit collusion and those in the legal literature. We provide examples of when the economic environment might be such that collusive profits can be achieved without communication and, thus, when tacit coordination is sufficient to elevate profits versus when communication would be required.

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†eug2@psu.edu, Department of Economics, Penn State University
‡rcm10@psu.edu, Department of Economics, Penn State University
§marx@duke.edu, Fuqua School of Business, Duke University
1 Introduction

In this chapter, we examine the economics literature on tacit collusion in oligopoly markets and take steps toward clarifying the relation between tacit collusion in the economics and legal literature. Economists distinguish between tacit and explicit collusion. Lawyers, using a slightly different vocabulary, distinguish between tacit coordination, tacit agreement, and explicit collusion. In hopes of facilitating clearer communication between economists and lawyers, in this chapter, we attempt to provide a coherent resolution of the vernaculars used in the economics and legal literature regarding collusion.¹

Perhaps the easiest place to begin is to define explicit collusion. It is our understanding that both the economics and legal professions use the term explicit collusion to mean an agreement among competitors that relies on interfirm communication and/or transfers to suppress rivalry.² In the United States, Section 1 of the Sherman Act makes explicit collusion illegal, forbidding agreements that unreasonably restrain competition and affect interstate commerce.³

As described in the seminal work of Stigler (1964), the key problem faced by firms attempting to collude is the need to deter secret deviations. The successful suppression of rivalry, elevating prices and restricting output relative to what it otherwise would be, creates incentives for secret price cutting by the firms. Thus, as described by Stigler, in order to successfully collude, firms must put in place collusive structures to govern the interaction among the colluding firms and between the colluding firms and other market participants. These

¹It is important to note that there is substantial debate within the legal profession on the meaning of these terms, as most recently noted by Kaplow (2011). See also Kaplow and Shapiro (2007) and Kaplow (2013). In this paper, in order to take steps toward clarifying communication between economists and lawyers, we opt for specific definitions of terms but recognize that there remain both substantial debate among legal scholars and latitude for interpretation by courts regarding these definitions.

²As we discuss legal concepts it should always be understood that it is our interpretation as economists, not as legal scholars, since we are not the latter.

³See Turner (1962). The Sherman Act states: “Every contract, combination in the form of trust or otherwise, or conspiracy, in restraint of trade or commerce among the several States, or with foreign nations, is declared to be illegal.” (15 USC § 1)
necessary structures include pricing, allocation, and enforcement structures. One would typically expect that the establishment and implementation of these structures would require communication and possibly transfers among the colluding firms, in which case the conduct would fall under the heading of explicit collusion.

However, firms in an oligopoly can be expected to recognize their mutual interdependence in the market. Each firm realizes that its profits depend not only on its own actions, but also on the actions of its rivals. It is possible that firms, each possessing this insight and understanding that its competitors all possess it, might be able to succeed in the implementation or even the establishment of a collusive agreement without communication. There has been broad agreement in principle that monopoly conduct can arise spontaneously in highly concentrated markets that satisfy some other (possibly restrictive) conditions. But, because economists have not yet been able to characterize those conditions with full confidence and precision, there has been room for courts to vary from one another. Since the 1980s, legal scholars have couched essentially this point in the language of an “extra ingredient” of centralization. As stated in (Kovacic et al., 2011):

In highly concentrated markets, the recognition of interdependence can lead firms to coordinate their conduct simply by observing and reacting to their competitors’ moves. In some instances, such oligopolistic coordination yields parallel behavior (e.g., parallel price movements) that approaches the results that one might associate with a traditional agreement to set prices, output levels, or other conditions of trade. The line that distinguishes tacit agreements (which are subject to section 1 scrutiny) from mere tacit coordination stemming from oligopolistic interdependence (which eludes

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4For elaboration on collusive structures, see Marshall and Marx (2012, Chapter 6). Levenstein and Suslow (2013) state, “While it has often been presumed that cartels’ demise results from cheating by member firms tempted by short term profits, empirical analysis suggests that cheating rarely destroys cartels. The potential profits from collusion provide sufficient incentives for cartels to develop creative ways to limit the temptations that inevitably arise.”
section 1’s reach) is indistinct. The size of the safe harbor that Theatre Enterprises recognized depends on what conduct courts regard as the “extra ingredient of centralized orchestration of policy which will carry parallel action over the line into the forbidden zone of implied contract and combination.” [Schwartz et al., 1983] Courts enjoy broad discretion to establish the reach of section 1 by defining this “extra ingredient” broadly or narrowly. (Kovacic et al., 2011, p.405)

In this chapter, we offer a way to distinguish between the legal profession’s use of tacit coordination and tacit agreement as in the above quote and to reconcile those with notions from the economics literature.

In order to parse between the language of the economics profession and the language of the legal profession, it will be useful to recognize that in order to collude, firms have to solve two broad problems: how to initiate a collusive arrangement and how to implement that arrangement. As described by Isaac and Plott (1981), “First an opportunity for conspiracy must exist. ... The opportunity to conspire must be followed by an attempt to conspire. The attempt to conspire must be followed by an actual conspiracy and the resulting conspiracy must have an impact on the market.” (Isaac and Plott, 1981, p.1)

The problem of initiating collusion involves coming to agreement on what the collusive structures required to deter secret deviations will be. This includes coming to agreement (reaching at least mutual knowledge, and perhaps common knowledge) regarding the mechanism for elevating prices, how the rents from collusion will be split among the firms, and how deviations will be detected and deterred. The problem of implementation involves managing the ongoing operation of the collusive arrangement, including the implementation of the collusive structures. The pricing structures may require coordination of price increases, the allocation structures may require transfers among the firms to achieve the agreed division of the collusive gain and to ensure incentive compatibility of the arrangement, and the enforcement structures may require information collection to stay in sync about the environment and to maintain
compliance.

Solving the two broad problems of initiation and implementation may re-
quire explicit communication among the firms. This is particularly true in
environments with strategic buyers. As noted by Isaac and Plott (1981, p.2),
“Markets have two sides, and those on the other side and not a party to the
conspiracy may not passively acquiesce to the establishment of such a conspir-
acy. The market reactions might be such that the efforts of the conspirators
are rendered ineffectual.”

Collusion without communication at the implementation stage would re-
quire that the firms establish during the initiation stage a contingent agree-
ment specifying reactions to observable outcomes during implementation,5 in
particular specifying what outcome would trigger reversion to noncollusive be-
havior (or, on a temporary basis, to even more severe retaliation). Depending
on the product, market, and industry, the set of contingencies may be too
numerous or too complicated for there to be the absence of communication
during implementation. However, in some environments, which we discuss
below, the problem of initiation or implementation may be solvable without
explicit communication or transfers.

Table 1 illustrates our interpretation of how the economics and legal pro-
fessions would classify coordination depending on whether there were commu-
nication or transfers and either the initiation or implementation stage. It is
important to note that this is one interpretation, and that both some legal
scholars and some courts may interpret the terms in different ways.6

5In the context of illegal agreements, there is no distinction between verifiable and ob-
servable, unless we consider a third-party cartel organizer with quasi-judicial powers (see

6“All concur that express agreements are a subset of interdependent behavior that counts,
that is, triggers liability. There is, however, no sharp consensus either on the boundaries of
this subset or on whether other subsets, such as one including tacit agreements, also suffice.
These and other questions are interrelated. For example, if express agreements are defined
broadly, to include what others might deem to be tacit agreements, then express agreements
might be viewed as exhausting the space of interdependent behavior that suffices, supposing
that the only other candidate behavior involves tacit agreements. Many, including the
Supreme Court in both earlier decisions and its most recent (discussed in Section III.B),
do in fact state that tacit agreements are sufficient, yet it is hard to know what to make of
these proclamations given the great ambiguity of the term.” Kaplow (2011, p. 700).
As described in Table 1, communication or transfers at the implementation stage is sufficient for either the economics or legal profession to classify the conduct as explicit collusion. The economics literature would also classify conduct as explicit collusion if there were only communication or transfers at the initiation stage; however, that is likely the case that the legal literature would term tacit agreement. With no communication or transfers at either the initiation or implementation stage, the economics literature would refer to the conduct as tacit collusion or implicit collusion (in what follows we use only tacit collusion), while the legal profession would likely refer to this as tacit coordination—it appears that tacit coordination in the legal profession refers to long-run recognized mutual interdependence among oligopolists that

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7 An industry with a history of collusion that has resumed collusive conduct after a hiatus may not need communication at the initiation phase.

8 Direct interaction that is disguised through the use of an intermediary, such as a trade association, would be included in the definition of direct interaction. “The communication among colluding sellers needed to insure successful price reporting may be indirect, in that it all proceeds through a trade association or statistical service, but it is none the less explicit.” (Kaysen, 1951, p.266) On this point, Kaysen (1951) cites the Maple Flooring Manufacturers Association discussed in 268 U.S. 563 (1925).

9 Kaplow (2011, p.700) notes that there is no clear consensus in the law as to the illegality of a “tacit agreement.” It is our impression that this speaks to the lack of agreement regarding the definition of “tacit agreement” in the law. Again, we are posing one interpretation to take steps forward in clarifying communication between lawyers and economists.

10 With the understanding that transfers would require communication, in some cases we limit our description to requiring only an absence of communication.
generates outcomes that exceed those that would be realized under myopic interaction (static Nash equilibrium) without any direct interfirm communication or transfers.

Adding to the complexity of parsing terminology, some economics literature focuses only on the implementation stage of a collusive arrangement. In those cases, the outcome may be described in the economics literature as tacit collusion, even though it may be clear that communication would be required in order to initiate the arrangement. In these cases, the label tacit applies only to the implementation stage, not to the collusive arrangement as a whole. This was recognized by Green and Porter (1984). Green and Porter analyze a collusive arrangement that does not require communication at the implementation stage, but they recognize the need for communication at the initiation stage, saying:

> It is logically possible for this agreement to be a tacit one which arises spontaneously. Nevertheless, in view of the relative complexity of the conduct to be specified by this particular equilibrium and of the need for close coordination among its participants, it seems natural to assume here that the equilibrium arises from an explicit agreement. (Green and Porter, 1984, p.89, n.5)

In the remainder of the chapter, we continue this discussion and provide examples. In Section 2, we consider when the economic environment might be such that collusive profits can be achieved without communication and, thus, when tacit coordination is sufficient to elevate profits versus when strong or weak explicit collusion would be required. In Section 3, we discuss the evolution of the theory of explicit collusion in the economics literature, particularly as related to the question of under what circumstances explicit coordination among sellers would be required during the operation of the cartel in order to achieve prices above a competitive level. In Section 4, we focus on the issue of coordination during the initiation stage. We conclude with Section 5, which comments on the role of tacit coordination in antitrust litigation.
2 Tacit coordination versus strong and weak explicit collusion

Antitrust practitioners turn to the industrial organization economics literature for guidance regarding the issues concerning initiation and implementation of collusive agreements among firms in an industry. In this light, we provide comments regarding the key economic results and link these results to the issues confronted by antitrust practitioners as they evaluate a product/market/industry for a potential collusive agreement.

In the economics literature, a standard folk theorem characterizes the set of equilibria of a repeated oligopoly game and shows that for sufficiently patient firms (or for sufficiently short delay between repetitions of the game), the set of equilibria includes strategy profiles that generate the monopoly outcome.\footnote{The first folk theorem (cf. Friedman, 1977) assumed perfect information and perfectly patient players. The next generation (e.g., Fudenberg and Maskin, 1986; Radner, 1986) assumed near perfect patience (i.e., a discount factor asymptotic to 1) and perfectly correlated information. The current state of the art (cf. Mailath and Samuelson, 2006) makes much less stringent assumptions about information, but continues be phrased in terms of theorems about the asymptotic approach to perfect patience. Current folk theorems strongly support the idea that a collusive arrangement can be operated in an environment that is not informationally rich if producers are sufficiently patient. Econometric studies (e.g., Porter’s (1983b) study of the JEC cartel), as well as information disclosed in court proceedings, show that various groups of producers have been sufficiently patient for successful operation.}

A folk theorem says that, in some environment, the problem of operating the cartel can be solved without setting up any ongoing, centralized mechanism of coordination—there is no need for explicit accounting, settlement, and enforcement.\footnote{For a discussion of the canonical model of weak explicit collusion in a repeated-game model of Bertrand price competition, see Choi and Gerlach (2013).}

However, folk theorems deal with the implementation of collusion, and have nothing to say about its initiation. The folk theorem itself does not address whether firms would choose to play the strategies that generate the monopoly outcome nor how firms might coordinate on those strategies. As stated in Ivaldi et al. (2003, p.6), “While economic theory provides many insights on the nature of tacitly collusive conducts, it says little on how a particular industry...
will or will not coordinate on a collusive equilibrium, and on which one.”

The economics literature addressing collusion without communication at the implementation stage typically considers repeated interaction among firms that allows the firms to maintain higher prices with the (possibly implied) threat that deviation would trigger retaliation. Retaliation mechanisms must be both credible and sufficient (see Ivaldi et al., 2003). For sufficiency, the reduced payoffs from the retaliation must be sufficiently large to deter deviations in the first place, and in order to be credible, it must be in the best interest of the firms to follow through on the retaliation following the observation of a deviation.

The folk theorem literature typically assumes away the problem of cartel initiation by characterizing the set of Nash equilibria without communication in the implementation stage (see, for example, Fudenberg and Maskin, 1986). In a Nash equilibrium each player chooses a best response to the strategies of the other players, which means identifying the set of Nash equilibria means essentially identifying outcomes that could arise in the implementation stage if firms were to coordinate on strategies that support that outcome in the initiation stage.

The gas station example of Carlton, Gertner, and Rosenfield (1997) fits within this framework—two firms compete by setting prices, where those prices are perfectly observable and can be adjusted instantaneously, and profit of each firm is determined by the two prices and a fixed demand curve. Carlton, Gertner, and Rosenfield (1997) note, in this environment, that one would not be surprised to find that tacit coordination (involving no communication) could support the monopoly outcome. Any deviations from monopoly pricing would be immediately observed and met by a response from the other firm.

However, such equilibria can be sufficiently complex that it is difficult to believe that firms could coordinate on a particular outcome without communication. As described in Stigler (1964), the central problem facing a cartel is

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secret price cutting by cartel members, so effective collusion requires that firm establish collusive structures, including pricing, allocation, and enforcement structures, in order to avert secret deviations.\textsuperscript{14} Thus, one would expect that collusion would at least require communication at the initiation stage in order to establish the necessary collusive structures, although it is possible the conspirators might not need further communication once the collusive structures are established.\textsuperscript{15}

In the gas station example of Carlton, Gertner, and Rosenfield (1997), no round of preliminary communication was used before the implementation stage, but it is a rare circumstance when that could be effective. For example, there may be several equilibria that provide payoffs in excess of the static Nash equilibrium, and formation stage communication may be needed to coordinate on a given equilibrium. Given the absence of implementation phase communication, the firms may need to select an equilibrium that is not payoff maximizing in order to avoid secret deviations, such as hidden loyalty rebates. Retail gas stations in Canada have been accused of strong explicit collusion in recent years, suggesting that the incremental elevation of prices at the pump required interfirm communication at the implementation stage. Excerpts from the announcements of the price fixing conspiracy investigations by the Canadian Competition Bureau are as follows:

During the investigation, the Bureau uncovered evidence of agreements between competitors to fix the price at the pump where gasoline was sold to consumers. The evidence indicated that participants carried out the conspiracy mainly by phoning one another firms observe the realized prices of their rivals. As long as this information is available without communication, these equilibria would be viewed as weak explicit collusion.\textsuperscript{14}See also Marshall and Marx (2012, Chapter 6).

\textsuperscript{15}Levenstein and Suslow (2013) state that, “The most successful cartels do not simply agree to a certain set of parameters. They create organizations or governance structures that allow them to address challenges that arise sequentially, expand the scope of the agreement, and provide flexibility in changing economic conditions. ... In the most sophisticated cartels, top level management sets overall cartel strategy, but the inherent uncertainty of the economic environment requires ongoing communication and decision making among operational employees.”
to agree on the price of gasoline and about the timing of price increases, contrary to the conspiracy provision, section 45 of the Competition Act.\textsuperscript{16}

Today’s criminal charges and guilty pleas are the result of an extensive Bureau investigation that found evidence that gas retailers or their representatives in these local markets phoned each other and agreed on the price they would charge customers for gasoline. The Bureau’s investigation into potential price-fixing in the retail gasoline market continues in the Southeastern Ontario market.\textsuperscript{17}

These investigations do not imply that there was no elevation of price relative to static Nash from tacit coordination, or even weak explicit collusion, prior to the explicit interfirm communication during the implementation phase. However, the investigations highlight that strong explicit collusion was thought to be incrementally profitable by the colluding firms relative to the tacit coordination or weak explicit collusion that they had functioned under prior to the use of implementation phase communication.

Furthermore, the buyers in the Carlton, Gertner, and Rosenfield (1997) gas station example are not players in the game and so have no ability to take actions that might disrupt the ability of the two firms to maintain their tacit coordination. When buyers are players, they have an incentive to pursue strategies that disrupt equilibria that allow the sellers to capture supra-competitive profits. Buyer resistance limits the ability of firms to maintain collusive prices through only tacit coordination because buyer resistance exploits the lack of communication, monitoring, and enforcement characterizing tacit coordination.

If prices are not observable and demand has at least a small random component, then one enters the environment where the combination of a need for equilibrium path punishments with asymmetry of information about market


outcomes has the consequence that perfect collusion is not possible without communication at the operation stage. Other changes in the environment reinforce the need for implementation-stage communication, including moving away from posted prices to, for example, competitive procurements and allowing buyers to be true players in the game.

Collusive equilibria in games with repeated interaction that are supported by reversion to noncooperative behavior or some other equilibrium punishment, theoretically speaking, do not require communication among the firms in the implementation stage. Equilibria such as the collusive equilibrium constructed in Green and Porter (1984) are discussed in the law literature as “oligopoly pricing” or as consciously parallel decisions of a few dominant sellers in an industry to maintain the same high noncompetitive price.\textsuperscript{18} The idea is that this type of behavior might arise “without overt communication or agreement, but solely through a rational calculation by each seller of what the consequences of his price decision would be, taking into account the probable or virtually certain reactions of his competitors.” (Turner, 1962, p.661)

In the environment of Green and Porter (1984), demand uncertainty prevents firms from being able to monitor perfectly the quantity choices of their rivals. Because of this, punishment periods are triggered in equilibrium. In this environment, even in the optimal collusive mechanism without implementation-stage communication, there will be punishment periods with low profits for the firms, and the periods of high profits may not yield profits as high as under strong explicit collusion. In an environment with quantity competition and demand uncertainty, the choice of the quantity targets in cooperative periods must balance the cartel’s desire to decrease those targets to increase payoffs in cooperative periods and the cartel’s desire to increase those targets to reduce the frequency of punishment periods, where payoffs are low. The more aggressive are the cartel’s quantity restrictions, the greater is the incentive for unilateral deviations, which, in this model, can only be prevented by making it more likely that such deviations will trigger a punishment period.

The analysis of collusive mechanisms without implementation-stage com-

\textsuperscript{18}Posner (1976, p.40) refers to this as “tacit collusion.”
munication in theoretical environments can provide valuable insights into collusive behavior. However, in few real-world environments would the informational requirements be met for there to be tacit coordination equilibria or weak explicit collusion equilibria that provide firms with the same level of profits as through strong explicit collusion, where communication occurs at both the initiation and implementation phase.\textsuperscript{19}

As described in Marshall and Marx (2012, Chapter 1.4), when purchases are sufficiently large and infrequent, or demand is sufficiently uncertain, or buyers are strategic, even though firms are engaged in repeated interaction over time, they may not be able to accomplish payoff-maximizing collusive outcomes.\textsuperscript{20} With enough lumpiness or randomness or difficulty interpreting responses from buyers, a firm often cannot rely on repeated play to discipline its rivals. Without communication and transfers, the sellers are left in a tough spot in terms of trying to achieve substantially elevated prices and profits. Secret deviations induced by self-interested profit maximization will creep into their conduct, and joint profits will fall short of monopoly levels.\textsuperscript{21} Strong

\textsuperscript{19}“Oligopolists behaving in a legal and consciously parallel fashion could achieve high and rising prices, even as costs remain stable, by engaging in price leadership. The odds that they could achieve a price and profit increase and maintain incredibly high incumbency rates – that is, maintain the very same distribution of municipal contracts year after year – are miniscule, however, unless the oligopolists were communicating with one another.” City of Tuscaloosa v. Harcros Chem., Inc, 158 F.3d 548, 565; (11th Cir. 1998). The clear presumption of the court in this case was that buyers, the municipalities in Stateplace Alabama, would conduct competitive procurements and push back against price increases by the sellers to the effect that incumbency rates would be volatile. It was not the presumption of the court that the buyers were passive. Rather, the court presumed that the buyers were players in the game, and thus much different from the customers of the gas stations in Carlton, Gertner, and Rosenfield (1997).

\textsuperscript{20}See Ivaldi et al. (2003) on how the feasibility of weak explicit collusion is affected by the number of firms, asymmetries among firms, entry barriers, market transparency, demand growth, innovation, product differentiation, multi-market contact, and other factors. See Choi and Gerlach (2013) on how the feasibility of weak explicit collusion is affected by the number of firms, symmetry and concentration, demand conditions, multi-market contact, imperfect observability and monitoring, incomplete information and communication, vertical mergers and restraints, and the presence of antitrust leniency programs.

\textsuperscript{21}The difficulties associated with sustaining successful tacit collusion when buyers make purchases infrequently or demand is uncertain can be seen formally in Tirole’s (1988, ch. 6.7.1, pp. 262–65) exposition of Green and Porter’s (1984) model of tacitly collusive behavior. (Marshall and Marx, 2012, Chapter 1.10)
explicit collusion gives the firms the additional tools of communication and interfirm transfers that may allow them to achieve the joint profit maximizing price.\textsuperscript{22}

\section*{2.1 Illustration}

In what follows, we describe the distinction between static competition, tacit coordination, weak explicit collusion, and strong explicit collusion using a series of examples. These examples are drawn from Chapter 1 of Marshall and Marx (2012).

\subsection*{2.1.1 Example 1: Observable posted prices}

Consider a small town where, on opposing street corners, there are two firms that sell the same product.\textsuperscript{23} These firms are the only two firms that sell this product in the town. Other firms who sell products exactly like this are at least 150 miles away from this town.

The firms can post a price at their location for everyone to see. Each firm can post a price exactly once a day at exactly the same time. Consumers buy from the vendor offering the best value. Absent any non-price competition between the firms, if the two firms post the same price, then they each get half of market demand.

\textbf{Scenario 1: Gas stations: Static environment} We begin by assuming the firms are two gas stations, and we label them as A and B. Suppose that each one of these gas stations buys its gasoline for $2 per gallon and that, because demand is noisy, each knows that a price somewhere between $2.90 and $3 per gallon maximizes their combined profits. In a static environment, where no firm was concerned about future profits, rivalry between the firms

\textsuperscript{22}\textcite{Levenstein and Suslow (2011, Table 3)} show that approximately one-third of the cartels they consider used a mechanism to arrange transfers when a cartel member’s sales exceeded its cartel allocation.

\textsuperscript{23}This example is based on Carlton, Gertner, and Rosenfield (1997).
implies that the price would be the marginal cost, or $2.01 if pricing is done in cent increments.

Scenario 2: Gas stations: Dynamic environment, uncertain demand
Now change this environment to allow more dynamic interaction. Suppose that the future matters almost as much as the present to each of the gas station owners. It is natural to think of the folk theorem, which suggests that the monopoly outcome should be possible without implementation-stage communication, but without communication what price would be selected between $2.90 and $3.00? Perhaps the two firms can settle quickly on a price such as $2.90 but with a small amount of communication at the initiation phase they may be able to agree to higher price, say $3. In this case, weak explicit collusion would allow the two firms to achieve a bigger payoff than tacit coordination.

Scenario 3: Gas stations: Dynamic environment, uncertain demand, non-price competition But suppose there are non-price dimensions for interfirm rivalry such as loyalty rebates, cleanliness of facilities, service for cars, and a variety of items available in the service station shop. If both firms are charging $2.90 through tacit coordination, but firm A starts to experience an erosion of its market share then firm A would infer that firm B is competing on non-price dimensions. Firm A can then either cut its price or increase non-price offerings.

In this kind of environment, with non-price competition available to each firm, it may not be possible for each firm to experience an increase in profits from an increase in price—each firm may invest in the non-price competition to an extent that no incremental profit is left from the price increase. But, the two firms could communicate at the initiation of collusive play and commit to the absence of non-price competition. If the market share of any firm falls significantly below 50% then they would know there was a breach in that the

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24 For empirical evidence of collusion in retail gasoline, see Borenstein and Shepard (1996). See also Slade (1992).
other firm was offering non-price enhancements implying that the agreement
would end and prices would revert to marginal cost. Each firm may recognize
that without communication during implementation it will not be possible to
maintain a price of $3—there will be just too much temptation for non-price
competition. But they may determine a lower price, say $2.15, where they are
each relatively confident that the initiation-stage communication regarding
assurances of non-price incremental enhancements will be honored, since the
payoff to a secret deviation is just too low (relative to a price of $2.90).

There may also be a lower price, such as $2.10, that the firms need no ini-
tiation stage communication to achieve. Specifically, the payoff to investment
in non-price enhancements is just too low when the price is $2.10 for either
firm to undertake it—a slightly higher price would induce such investments in
the absence of initiation-stage communication.

In this example, tacit coordination at $2.10, weak explicit collusion at
$2.15, and strong explicit collusion at $3 are each conceptually possible.

The price increment that can be achieved with weak explicit collusion de-
pends on the magnitude of the investment each firm would need to make in
secret non-price actions to deviate without direct detection. If the investment
is small, then the price increment above $2.10 that can be achieved with weak
explicit collusion is small. Similarly, if the investment is large then the price
increment will be much larger.

2.1.2 Example 2: Unobservable bid prices

We now change things again. Suppose that the two firms are not gas sta-
tions but instead are manufacturers of plastic industrial bags used for packag-
ing a number of different types of finished products.

If you think about a plastic bag that is used for packaging some types of
processed vegetables, fertilizer, or ready-mix concrete, there will be printing
on the bag. Many firms that package and ship their product, or are engaged
in the business of shipping, buy plastic industrial bags. There are standard
bags constructed of standard materials with no printing. An example might
be an 18×12 inch 3ml thick bag. However, these kinds of bags are typically a
small percentage of sales for an industrial bag plant. Rather, most sales are to manufacturers or shipping companies that buy large numbers of bags of specific sizes, where the material used in the bag construction, the thickness of the bag, and the printing on the bags are specified by the buyer.

Almost all of the purchases of such bags will be done by procurement. Specifically, a buyer will invite bag plants to submit bids, where the details of the bag size/thickness, material used, and printing will be part of the bid solicitation. The bag plants may post a price for the aforementioned 18×12 inch standard bag and may offer such bags at the posted price in small volumes at each firm’s office location, but the bulk of sales for each bag plant will be through procurements. The posted price for a standard bag has no effect on the bid submitted by any bag plant in a procurement, and the buyers know that.

Each bag plant has a reasonably good idea of the costs of its rival. Also, unlike the gas station example, where no consumer is going to drive 150 miles each way to buy gas from a cheaper nonlocal gas vendor, it is economically viable to buy bags from a bag plant that is 150 miles away, although such a bag plant has additional transport costs. The bid by any bag plant in any procurement is not revealed by the buyer to the competing firm—a bag plant only knows if it won or lost any given procurement. For this example, we assume that each bag plant values the future highly.

If we consider an environment with just large buyers, the two bag plants can potentially agree on bids to submit at the initiation phase to make sure that the work is shared in a relatively equal way. If one bag plant has won a recent large contract, then the other bag plant can win the next one. The two bag plants can potentially develop initiation phase contingent plans about the bids each will submit. Thus, the behavior can be supported with weak explicit collusion.

**Scenario 4: Bag plants: Small number of large strategic buyers, uncertain demand** To answer the question as to why strong explicit collusion might be required above and beyond weak explicit collusion, suppose that
buyers are small in number, large in size, and strategic. In addition, assume demand is uncertain. Suppose that the large buyers conduct procurements infrequently and at irregular intervals. Suppose that each bag plant expects to receive, in total, one bid solicitation each quarter. Furthermore, suppose that these large buyers may choose to extend their current bag contract terms for a year or more without reconducting a procurement. There are not many contract awards each year—on average, only 4 per year. If one bag plant receives only one or even none of the awards, then it will be in dire straights.

For a given procurement, suppose that a bag plant submits a bid that is close to the joint monopoly price and observes that it loses the award. This could be from one or more of four causes: (i) random bad luck, (ii) its competitor undercut it in the bidding, (iii) there was a negative demand shock in the market that affected both bag plants but the shock was unknown as such to either bag plant, and (iv) the buyer acted strategically by offering the contract award to a competitor with extended award length if the bid price were lowered by that firm. Suppose the bag plant in question loses a second procurement. Now the bag plant that has not won a contract in 6 months is really feeling the financial pinch.

Strong explicit collusion can resolve the conundrum for the two firms. By communicating and monitoring one another during the implementation phase the firms are able to eliminate certain conjectured causes for the lack of an award and directly address buyer resistance.

Weak explicit collusion is much more difficult to implement. The number and complexity of contingencies that would need to be specified at the initiation phase is substantial. Creating contingent plans to address the entire range of potential buyer resistance would be like creating a contingent plan for a chess game.

The results of Green and Porter (1984) imply that even a minor incremental payoff from tacit coordination would not be possible. When a procurement environment has this much noise from a number of sources, direct interfirm communication at both the initiation and implementation phase is needed to suppress interfirm rivalry.
Scenario 5: Bag plants: Many non-strategic buyers, uncertain demand

Consider another scenario where there were many small manufacturers buying bags with great regularity. Suppose that these buyers are all in the same business. For example, they make frozen French fries. They need bags to ship their frozen French fries. But suppose that frozen French fry demand for these local manufacturers is highly unstable, and the instability affects each of them in a similar way. Suppose that the bag plants cannot observe frozen French fry demand conditions. Furthermore, each bag plant cannot observe how much production is occurring at its rival’s plant. In normal demand conditions, each bag plant expects to receive 500 bid solicitations each quarter. But a given frozen French fry manufacturer may decide not to make any award in a given quarter if demand conditions are sufficiently depressed.

Suppose that a bag plant submits bids over the course of a quarter that are at the joint profit-maximizing level. If it observes that it has won approximately 250 of 500 solicitations, then it will continue to hold to that price. But, if it observes that it has won 125 of 500 solicitations, then the bag plant has some serious questions to address. Are the frozen French fry manufacturers experiencing a negative demand shock, or has the other bag plant undercut the joint profit-maximizing price to obtain more awards, or is some of both occurring?

By assumption, the bag plant in question is unable to determine the cause for the reduced number of contract awards. As a consequence, a bag plant will reduce its bid below the joint profit-maximizing price and move it toward the marginal cost of production. To see this, note that if the bag plant does not react based on the belief that bad outcomes are attributable to negative demand shocks and not the conduct of its rival, then the rival would take advantage of this belief by undercutting the high-priced bag plant.

In this case, strong explicit collusion can elevate profits, probably to the joint profit-maximizing level. But, weak explicit collusion can produce a payoff above static Nash albeit below strong explicit collusion. Initiation-stage communication can resolve the price that each firm will bid at the procurements. The weak explicit collusion described here is the same as that in Green and
2.2 Context for merger reviews

The issues of tacit and explicit collusion also arise in the context of merger reviews. The 2010 Horizontal Merger Guidelines (Guidelines) of the U.S. Department of Justice and Federal Trade Commission discuss concerns about “express collusion,” by which they presumably mean strong explicit collusion, and also about “coordinated interaction.”25 According to the Guidelines (pp.24–25):

Coordinated interaction includes a range of conduct. Coordinated interaction can involve the explicit negotiation of a common understanding of how firms will compete or refrain from competing. Such conduct typically would itself violate the antitrust laws. Coordinated interaction also can involve a similar common understanding that is not explicitly negotiated but would be enforced by the detection and punishment of deviations that would undermine the coordinated interaction. Coordinated interaction alternatively can involve parallel accommodating conduct not pursuant to a prior understanding. Parallel accommodating conduct includes situations in which each rival’s response to competitive moves made by others is individually rational, and not motivated by retaliation or deterrence nor intended to sustain an agreed-upon market outcome, but nevertheless emboldens price increases and weakens competitive incentives to reduce prices or offer customers better terms. Coordinated interaction includes conduct not otherwise condemned by the antitrust laws.

Thus, it appears that the Guidelines intend for the term “coordinated interaction” to encompass conduct ranging from tacit coordination to strong

explicit collusion and apparently more. For an attempt to disentangle the notion of parallel accommodating conduct, which is introduced in the 2010 revision of the Guidelines, from existing notions of collusion, see Harrington (2012).

3 Evolution of the theory of explicit collusion

Before the arrival of formal dynamic game theory, ideas on collusion without communication relied on two theories: focal points and price leadership. The theory of focal points stated in Schelling’s (1960) *The Strategy of Conflict* played an important role in the justification of tacit coordination as shown in Scherer (1970, p.192): “Even a price that has no particular uniqueness or compulsion in its own right may become a focal point simply by virtue of having been quoted repeatedly.” On price leadership, when Bain (1968) analyzed conventions and agreements to set a price leader he argued that:

Evidence of such direct consensual action not being found (and it seldom is), it is more usual to recognize price leadership as a form of tacit collusion, resulting from the existence of an unspoken agreement. The notion of tacit agreement, however, is itself somewhat nebulous, and it seems perhaps equally appropriate to designate the conduct pattern in question as one of interdependent seller action without basis in agreement. (Bain, 1968, p.312)

From this starting point evolved the analysis of collusion in terms of repeated and dynamic games using the tools of game theory.26 The focus was on the tension described by Bagwell and Staiger (1997, p.82) as: “Collusion is a balancing act. Each colluding firm balances the short-term temptation to cut its price against the expected long-term cost of the price war that such an act might instigate.” In this literature, the threat of punishment of deviations

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26See Choi and Gerlach (2013) for a review of experimental evidence on collusion and antitrust leniency programs.
is the force driving collusive outcomes. Naturally, the ability of firms to detect deviations plays an important role.

At the same time as one community of economists was developing the relatively informal theories of focal points and price leadership, another community was developing explicit, formal theories of equilibrium in repeated games. However, after having flourished for more than a decade, by the mid 1960s game theory was viewed by many economists as a sterile mathematical field that was too highly schematic to be applied successfully to collusion or to other concrete issues of economics and social science. Large credit for superseding that negative view is due to James W. Friedman. Both through his own research and through his lucid presentation of the game-theoretic ideas in *Oligopoly and the Theory of Games* (1977), Friedman educated “mainstream” IO economists about the way of thinking and the specific results and techniques developed by game theorists in the 1960s and 1970s. One of his important, specific contributions was to frame repeated games in a discounted-payoff context, rather than using the limit-of-payoffs criteria that had dominated preceding research. By doing so, he brought into focus the crucial role of the trade-off between short- and long-term costs and benefits that Bagwell and Staiger’s summary emphasizes.

The main limitation of Friedman’s theory of collusion without implementation-stage communication was its restriction of attention to an environment with perfect monitoring. Abreu (1986) provided the capstone of that research program by characterizing the maximum profits that can be sustained without implementation-stage communication or transfers in terms of the most severe punishments for defection that colluders could impose on one another.

The literature advanced to consider the possibility of collusion without implementation-stage communication, i.e., weak explicit collusion in our terminology, in ever more complex environments. For the case of imperfect monitoring, Green and Porter (1984) show that some degree of collusion can be achieved in a quantity setting supergame even when firms are not able to perfectly observe their opponents actions. Abreu, Pearce and Stacchetti (1986)
study the same issues as Abreu (1986) but for the imperfect monitoring case. Brock and Scheinkman (1985) and Lambson (1994) study weak explicit collusion in price setting supergames. The first shows that some degree of price elevation can be achieved and study how this is affected by the number of firms in the industry. The second studies the maximum profits achievable through weak explicit collusion and shows that the structure of the most severe (optimal) punishments is much simpler in price setting supergames than in quantity setting supergames. Finally, Benoit and Krishna (1987) study weak explicit collusion in a model in which firms choose both quantities and prices.

Additional complexities are considered in literature that considers environments with incomplete information. In environments with incomplete information, belief-free equilibria are important for private monitoring. If players observe different signals—for example, an oligopolistic market in which each firm observes only its profits but no prices, quantities or other firm’s profits—then beliefs about what other players are observing complicate the equilibrium analysis. But still, given that the signals are informative enough, it is possible to sustain some level of collusion with a subset of subgame perfect equilibria in which beliefs about what the other players have observed play no role, the belief-free equilibria.\footnote{Abreu, Pearce and Stacchetti (1990) rely on a different method to solve for the equilibria of dynamic games, using the notion of self-generation, which allows applications to asymmetric games.}

In environments with incomplete information about the profitability of a market, recent work shows that collusive payoffs may still be possible without communication at the implementation stage. See, e.g., Yamamoto (2012) and Schenone (2011).\footnote{See Ely and Valimaki (2002) and Ely, Horner, and Olszewski} In this literature, in some cases, the necessary conditions
for folk theorem type results are restrictive, telling us that a folk theorem does not hold for a wide range of games.

Additional challenges to the legal interpretation of firm behavior and enforcement of antitrust laws are raised by more recent literature such as Horner and Jamison (2007), which presents a model in which almost no information is required to sustain full collusion. The model consists of an infinitely repeated price-setting game with inelastic demand. At each period, each firm draws a per unit cost that can be either high or low. Costs are private information. At each period, the firm with the lowest price gets the whole demand and if more than one firm set the lowest price a randomly selected firm gets the whole demand. A firm only observes its own price and whether it sells. In this model, full collusion is understood as an equilibrium in which in almost all periods a low price gets the sale and it charges a price close to the monopoly price. The main result states that if firms are patient enough, they can get arbitrarily close to the monopoly outcome without implementation-stage communication.

4 The initiation phase of collusion

Prior to this point, our discussion has focused primarily on the question of under what circumstances explicit coordination among sellers would be required during the operation of the cartel, in order to be successful in maintaining prices above a competitive level. However, we have emphasized that coordination might also be necessary prior to the cartel beginning to operate, in order to agree on a mechanism by which subsequent collusion would be enforced and on how the rents from successful collusion would be distributed among the cartel’s members. We now turn to the questions: under what conditions would such explicit coordination/communication ex ante be required,
and under what conditions would it even be helpful, in order for a viable cartel to be born? Throughout most of our investigation, we maintain the view taken in the several preceding sections that industry conduct is a Nash equilibrium outcome of strategic interaction between agents in the market environment.

### 4.1 Nash equilibrium as self-enforcing agreement

A widely held interpretation of a Nash equilibrium, is that it is a self-enforcing agreement among the players of a game. (Myerson 1991) Taking this interpretation literally, a Nash equilibrium could not be tacit coordination, since an agreement cannot be reached without communication.

A less restrictive interpretation is that players conduct themselves as though they had previously reached an agreement among themselves, and moreover the counterfactual agreement would enforce itself. That is, the prior meeting to reach and ratify an agreement can be treated as a parable rather than as an actual historical event, much as the political theorists interpret the ‘social contract.’ Aumann and Brandenburger (1995, pp.1162–1163) have made explicit what such a counterfactual account of agreement entails:

Suppose that the game being played [by two players] (i.e., both payoff functions), the rationality of the players, and their conjectures [about the probabilities with which actions their respective opponents will take] are mutually known. Then the conjectures constitute a Nash equilibrium. . . [Moreover, supposing that there are more than two players, that] their payoff functions and their rationality are mutually known, and that their conjectures [about one another’s actions] are commonly known. Then [their conjectures coincide and constitute] a Nash equilibrium.30

To say that something is mutually known, just means that everyone knows it. To say that it is commonly known, means that everyone knows it, everyone

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30 Because we will not discuss it here, we have omitted a common-prior assumption of Aumann and Brandenburger’s result.
knows that everyone else knows it, and so forth. People generally do not need to meet, in order to have mutual knowledge of some aspect of their shared environment. For example, if ten people read this sentence, then they have mutual knowledge that it begins with a three-letter word. But, unless they are all read the sentence in one another’s presence (in which case, they are meeting) or they subsequently discuss the sentence after having read it (in which case they also meet, subsequently to their reading), presumably none would know that the others know this fact.

Moreover, although direct observation (without communication) by all competitors is sufficient to establish mutual knowledge of an observable fact, it is not sufficient to establish mutual knowledge of one another’s intentions. In particular, given that repeated games of the sort considered above have both competitive and collusive equilibria, one competitor cannot observe which equilibrium another expects everyone else to play (and so will play himself). Thus, in any cartel situation—even a duopoly—it is difficult to see how prospectives would achieve mutual knowledge of each other’s intent to collude, unless they were to communicate those intentions to one another. Common knowledge, and mutual knowledge about other players’ mental states (e.g., intentions, beliefs) is called higher-order knowledge.

So, let’s formulate a two-part Need to Meet (NTM) principle:

1. In general, players cannot arrive at a Nash equilibrium by a process of reasoning, unless they attain higher-order knowledge of one another’s conjectures. In particular, they cannot “reason their way” to a collusive Nash equilibrium without attaining higher-order knowledge.

2. Such higher-order knowledge cannot be attained, in practice, without having an explicit meeting.

Let us acknowledge, before going further, that the NTM principle is not a logical consequence of Aumann and Brandenburger’s result. They have framed a sufficient condition for Nash equilibrium, while the NTM principle envisions common knowledge, and hence occurrence of a meeting, as a necessary condition. They write (p.1163) that “It is always possible for the players to blunder
into a Nash equilibrium ‘by accident,’ [but a higher-order-knowledge assumption] cannot be... significantly weakened.” 31 But, attaining such higher-order knowledge is virtually a necessary prerequisite for players (e.g., sellers in a market) to “reason their way” to any particular type of Nash conduct, collusive or otherwise. Henceforth we will treat it as being necessary. That is, we stipulate clause (1) of the NTM condition.

If clause (1) of the NTM condition holds, then clause (2) is tantamount to an assertion that higher-order knowledge of competitors’ intent to collude must always be reached explicitly, rather than tacitly. In the next two sections, we assess clause (2).

4.2 What does “arrive by reasoning” mean?

We have identified a situation in which Aumann and Brandenburger’s higher-order-belief conditions are satisfied, and in which there is a self-confirming conjecture that a particular Nash equilibrium will be played, with a situation in which the players have arrived by reasoning at the equilibrium. This situation is contrasted with the one in which, in Aumann and Brandenburger’s words the players have blundered into an equilibrium by accident. But to make this identification is of no help for assessing whether or not clause (2) is reasonable. To make that assessment, we need an explicit account of the process that players are envisioned to follow. That account is provided by the theory of rationalizable play. A player is rational, if he plays an action that is a best response to his conjecture (represented by a probability measure over opponents’ possible actions) of what actions other players may take. The first step of his reasoning process is to eliminate actions that would not be a best response to any conjecture about what others might do. Then, he would put himself in the shoes of other players, whom he knows to be rational. He would realize that, at a minimum, each of them would also be eliminating any never-a-best-response actions. At that point, the player would review his actions that have survived being eliminated in the first round. Has any

31 Aumann and Brandenburger (1995, sec. 5) show that their result would fail if any of the conditions were weakened significantly.
of those actions been retained solely because it would be a best response to some opponent’s action that is never a best response for the opponent? In that case, he knows that the opponent will not play the action in question, so he should eliminate the action that he retained on account of it. When each player has gone through this second round of eliminating actions, and has again put himself in the shoes of other players who have done likewise, there will be a third round, and so on. A player’s actions that survive elimination in every round, forever, are rationalizable for that player. Pearce (1984) and Bernheim (1984) formulated the concept of rationalizability, and they showed that all actions played (with positive probability) in a (mixed-strategy) Nash equilibrium must be rationalizable. Tan and Werlang (1988) showed that a profile of actions in a game is rationalizable for the respective players if and only their being played is consistent with it being common knowledge that all players are rational.\footnote{Again here, we are giving a brief incomplete statement of a result in order to avoid digression from the topic of the chapter.}

That is, the iterative process of reasoning by which players determine their rationalizable actions (a) never eliminates any action that is ever played in any Nash equilibrium, and (b) always eliminates any action that would be inconsistent with common knowledge of rationality.

In view of these results, rationalizability is evidently the right concept to make precise the idea of “arrival by reasoning” at Nash play in a game.

4.3 The (im)plausibility of arriving at collusion by reasoning

The standard, intuitive view of tacit coordination in the IO/legal literature seems to be as follows. At the beginning of the story, each of prospective colluders is pondering whether to act competitively or monopolistically. These agents do not communicate with one another. Nevertheless, it is common knowledge that all of them want to maximize their respective profits, and it is also common knowledge that there is a self-enforcing profile of actions such as has been discussed above in Section 2.1, which would solve the problem of
maximizing joint profits and distributing them equitably (according to some standard that they commonly know that they all embrace) subject to the constraint that collusion must be self enforcing. In this situation, according to the intuitive view, each agent should conjecture that the other agents will play as specified in that profit-maximizing equilibrium, and therefore should also play according to the equilibrium because the equilibrium is a self-enforcing arrangement.

This statement of the intuitive view is ambiguous between two versions. One version—let’s call it the convergent-elimination view—assumes that, bootstrapping from just the facts that have been stipulated to be common knowledge, the iterative elimination procedure will “zero in” on precisely the profit-maximizing Nash equilibrium. That is, that equilibrium profile of actions will be the unique profile of actions that survives the iterative-elimination process. The other version—let’s call it the focal-point view—recognizes that the rationalizable actions are those that could be chosen by agents with arbitrary beliefs (subject only to having certainty of the facts that are stipulated to be common knowledge) and makes a further assumption that all of the agents’ beliefs place very high prior probability on the profit-maximizing equilibrium being played. On this view, the role of the iterated-elimination process is to condition probabilities repeatedly on higher-order-belief conditions, raising agents’ probability assessments from the initial, very high levels to virtual certainty that the profit-maximizing equilibrium will be played.

The explication of “reasoning to an equilibrium” as rationalizability shows that the convergent-elimination view is completely untenable. To begin, the game among prospective colluders always has a competitive Nash equilibrium, as well as possibly having a collusive one. Property (a) of rationalizability is that no action consistent with any Nash equilibrium is ever eliminated. Thus, at most, rationalizability might establish that some Nash equilibrium or other might be played, but it could not establish that the collusive equilibrium specifically would be played. Further analysis is even less favorable than that to the convergent-elimination view. Specifically, rationalizability is such a weak solution concept that it is consistent with it being common knowledge
that no player will play a Nash strategy.\textsuperscript{33} Convergence to Nash equilibrium in general, let alone to a specific Nash equilibrium of a multiple-equilibrium game, cannot be achieved entirely from the “Archimedean lever” of common knowledge of rationality.

The focal-point view is logically coherent, because it adds a strong hypothesis about the prospective colluders’ prior beliefs to the minimal hypothesis of common knowledge of rationality. The focal-point view is as plausible, but only as plausible, as is that hypothesis. The gist of the argument for the hypothesis is that, when players of a game share a common background, and when that common background is common knowledge among them, then their conjectures are much closer to being coincident than would be the case if they lacked such shared experience.\textsuperscript{34}

Consider the plausibility of the focal-point view with respect to the spectrum of cases of collusion described in Section 2.1. Three questions should be considered:

1. Is it plausible that all prospective colluders would conjecture that some Nash equilibrium would be played?

2. If so, then is it plausible that they would all conjecture that some collusive equilibrium (that is, one that would achieve an above-competitive level of profit) would be played?

3. If so, would their conjectures all be (nearly) identical, so that they collectively would succeed in playing a specific, collusive equilibrium?

\textsuperscript{33}Bernheim (1984, pp. 1011-12) provides an example of a game that has a unique Nash equilibrium. It is a pure-strategy equilibrium in which each player plays one specific action with certainty. However, that Nash action is not the player’s only rationalizable action. To the contrary there is a set of profiles of actions, in each of which all players play non-Nash actions, such that rational players can have common knowledge of those profiles as conjectures.

\textsuperscript{34}The focal-point view was first set forth by Schelling (1960). Schelling envisioned two people who have common knowledge that they need to meet in Manhattan, but have not had an opportunity to communicate about where they should meet. He suggested that two tourists would both go to the Empire State Building, while two NYC natives would both go to Grand Central Station. That is, a common background makes a particular Nash equilibrium to be focal, but which equilibrium were to become focal would depend on specifics of the background.
Let’s stipulate for now that the answer to questions (1) and (2) are affirmative. Certainly the predominant economic modeling strategy in IO is to assume Nash play. We will further consider question (2) below, but at the very least, a collusive equilibrium is a natural conjecture in a context where it is common knowledge that everyone wants to maximize profits.

In our basic example of gas-station duopolists, affirmative answers to questions (1) and (2) strongly suggest an affirmative answer to question (3), as well. This implication reflects our assumptions in the example, such as that gasoline is an undifferentiated product, that competitors have no private information about their respective costs, and that price discrimination of any sort is impossible for any seller to implement. It is not quite true that there is a unique profit-maximizing equilibrium in this setting, but there is a unique price-maximizing equilibrium path, and it is a deterministic path. It does not matter much, then, what kind of punishments the colluders believe would be imposed off the equilibrium path. In fact, as long as each colluder conjectures that he would be punished with sufficient severity if he were to break the cartel, the equilibrium path of play will be realized. This outcome is robust to that type of heterogeneity of conjectures.

But, the farther one moves away from the basic example, the less plausible it is that industry participants would share a focal point. Consider three representative reasons why coordination would be unlikely to be achieved in various situations:

- In a differentiated-product oligopoly, joint profit maximization is no longer salient. Rather, there is a Pareto frontier of profit-maximizing price profiles. Because of imperfect substitutability, a seller can be better off to maintain a price somewhat above its competitor’s price than to sell at the competitive price. A firm that charges a high price, but one that is slightly lower than its competitor’s price, gets the lion’s share of the rents. The division of rents would be determined by the competitors’ respective bargaining power against one another. If each competitor believes that it is in the strongest bargaining position, for example, then the various competitors’ conjectures will correspond to different points
on their Pareto frontier of profitability, and coordination on a focal Nash equilibrium will fail.

- Even if competitors agree about what is their desired imputation of rents, there are a plethora of ways to achieve it. Consider, for example, the notorious cartel of electrical-generator manufacturers in the 1950s. Bidding in procurement auctions was rigged by a scheme that selected a winner according to the phase of the moon on the day bids were requested. As long as purchasers remained unaware of the arrangement, the lunar phase would be random, so the selection of the winning bidder would be equitable. But, certainly it could not have been common knowledge, prior to any communication, that this was the scheme that would be adopted. That is no more a focal point than using the last 2 digits of the number (mod 4) of shares traded on the New York Stock Exchange on the day prior to the opening of bidding. Or, one month prior to the opening of bidding. Or, the high temperature (mod 4) in Chicago on that day, as reported by the U.S. National Weather Service. The number of equally salient candidates for a coordination device is huge. To implement a Nash equilibrium, it is not sufficient for cartel participants to agree that they will split the market in a way that provides equal shares in the long run. They must have a unanimous conjecture about precisely how the splitting will be done.

- In an environment with private information, the equilibrium path will be stochastic. If “punishment phases” are randomly triggered by outcomes such as steep decline in demand at the cartel price, for which a deviation from collusion has a high likelihood-ratio statistic (even though deviation is not believed to have actually occurred), then colluders face a coordination problem. If the “punishment” is not going to last forever, then all colluders must abandon it simultaneously. The reason is that, while anyone acts as prescribed in that phase, prices tend to remain low. Thus, if someone continues that conduct for longer than the others conjecture, then they will perceive the resulting low price to be occurring
during the “active-monopoly” phase of the cartel and will initiate the punishment phase. With that phase being in force most of the time, the cartel will not be profitable. But there are many equilibria corresponding to various rules about when to re-enter the active-monopoly phase, none of which is an obvious candidate to be a focal point. One candidate, the profit-maximizing equilibrium, involves successive punishment phases of independent, random duration. (Abreu, Pearce and Stacchetti, 1986) Analogously to the problem of random selection of the respective winning bidders in a sequence of rigged auctions, there is indeterminacy in the selection of the randomizing device, and coordination will fail unless all participants adopt the same device.

To summarize, there are two versions of the idea that sellers in a market might coordinate on a specific pattern of collusive conduct without the need to communicate plans with one another. One version, that such coordination would occur simply as a result of the sellers following the logic of optimization in their shared situation—that is, the idea of “reasoning to an equilibrium”—does not survive careful game-theoretic analysis. The other version—the focal-point view—is logically sound. Nevertheless, because most market environments are sufficiently complex that there are numerous possible ways to collude, none of which will work unless it is adopted by all of the significant market participants, that view suggests that it is difficult, and probably rare, for successful collusion to obtain in the absence of explicit communication. The exception to this generalization is a market that is so simple and transparent—such as the gas-station-duopoly example discussed earlier—that there is a unique candidate for the optimal collusive agreement.

4.4 Risk dominance as an obstacle to collusion

Let’s further consider an infinite-horizon version of the gas-station duopoly. Suppose that the stations are identical, having a fixed cost $F$ of operation and selling gas at constant marginal cost. Moreover, suppose that there are only two prices that could be charged: a high $p_H$ and a low $p_L$. 

Let’s make an assumption that the gas stations will continue to charge $p_H$ forever if they both charge that price initially, but that they will both charge $p_L$ subsequently if they do not both charge $p_H$ the first time. This assumption expresses, in stark form, the idea that initial success in collusion establishes a successful long-term arrangement, but that any convergence to collusion after an initial failure to collude would be slow enough that the present discounted value of eventual collusive profits would be low.

If both sellers charge $p_H$, then each receives half of the monopoly profit $M$ in each period, and the cartel continues forever. Thus, the discounted present value of participating in the cartel is $\rho M$, where $\rho > 1$.\(^{35}\) If they both charge the low $p_L$, then each earns zero profit.\(^{36}\) If they start out making opposite choices, then the high-price station initially sells no gas but bears its fixed cost, losing $F$, while the low-price station initially earns revenue $p_L D(p_L) = 2F$ and consequently earns positive profit $F$ net of its fixed cost. Subsequently to the initial period, by the assumption above, both stations charge $p_L$ and earn zero profit. These payoffs are summarized by the following matrix.

$$
\begin{array}{cc}
  \langle p_H, p_H \rangle & \langle p_H, p_L \rangle \\
  \langle p_H, p_L \rangle & \langle p_L, p_L \rangle \\
\end{array}
$$

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If the interest rate is low (so that $\rho$ is large), $M$ is large, and $F$ is small, then $\rho M/2 > F$, and both symmetric-price profiles are Nash equilibria, with the collusive one (that is, for both sellers to charge $p_H$) being the more profitable one. Profitability seems to make collusion a focal point. However, there is a countervailing argument. A seller who sets price $p_H$ will suffer a loss if,

\(^{35}\)If the duopolists face 1-period interest rate $r$, then $\rho = (1 + r)/r$.

\(^{36}\)So, if $D(p)$ is market demand at price $p$, then $p_L D(p_L)/2 = F$. Note that, as pointed out by Edgeworth, this price would not be a Bertrand equilibrium (and, with positive $F$, no Bertrand equilibrium would exist) if a seller could marginally lower its price and capture the entire market.
contrary to his conjecture, the other seller charges \( p_L \). In contrast, a seller who sets price \( p_L \) will make an unanticipated profit if, contrary to his conjecture, the other seller charges \( p_H \). In this sense, setting \( p_L \) is the less risky choice. Indeed, if a seller conjectured that the other seller was equally likely to charge \( p_L \) or \( p_H \), then he would charge \( p_L \) in order to maximize his expected profit.

The technical name for this property of the low-price equilibrium is *risk dominance*. It was introduced by Harsanyi and Selten (1988), who proposed the *tracing procedure* as an account of how rational players choose among multiple Nash equilibria of a game that they are playing. Roughly speaking, their proposal was first to discard all but the Pareto frontier of the Nash equilibrium set, and then to use risk dominance to select among the remaining equilibria. But Aumann (1990) subsequently argued, in effect, that such a lexicographic priority for Pareto efficiency was misguided, and that risk dominance is actually the more compelling criterion. Harsanyi (1995) accepted Aumann’s argument and formulated a new equilibrium-selection theory based solely on a version of risk dominance.

The case for the risk-dominant equilibrium being played is even further strengthened by research of Carlsson and van Damme (1993). They consider games such as the one above, to which they add the realistic consideration that players’ information about one another’s payoffs is accurate but not perfectly precise. Imagine, for example, that two gas-station owners are franchisees of different chains and are contractually obligated to use their respective franchisers as their sole suppliers of gasoline. Each supplier sets its price at any date in a way that closely reflects the market price of crude petroleum, and each gas-station owner can read the price of crude in the newspaper but cannot directly observe the wholesale refined-gas price of his competitor’s supplier. Thus, each owner is highly confident that the other purchases gas at a price very close to his own price, but does not know his competitor’s price—or, consequently, his competitor’s payoff function—with complete accuracy. The generalization of Nash equilibrium to such a situation is *Bayesian Nash equilibrium*, and Carls-

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37 See Harsanyi and Selten (1988) for a formal definition of risk dominance and discussion of its significance.
Carlsson and van Damme prove that the risk-dominant Nash equilibrium of the complete-information game corresponds to the unique Bayesian Nash equilibrium of the incomplete-information version of the game, even as the precision of information tends to certainty.\(^{38}\)

Carlsson and van Damme’s formal proof is a piece of mathematical analysis that may seem to be a magician’s hat trick, but they provide a discussion that makes it seem much more intuitive. Recall the distinction between mutual knowledge and common knowledge that was introduced above. Something is mutual knowledge among a group of competitors if each knows it. A synonym for mutual knowledge is first-order knowledge. Define the competitors to have second-order knowledge of some fact if each of them both knows the fact itself and knows that the others know it. Third- and higher-order knowledge are defined successively in this way. A fact is common knowledge if there is first-order and second-order and... \(n^{th}\)-order and... knowledge of it. When information is sufficiently precise, it can be \(n^{th}\)-order knowledge (for very high \(n\)) that rational competitors will charge the monopoly price (that is, will play the payoff-dominant but risk-dominated strategy), but that is never common knowledge unless information about competitors’ payoffs is absolutely precise. Rubinstein (1989) has shown that no finite level of belief is sufficient to make it rational for everyone to play the payoff-dominant Bayesian Nash equilibrium. Only common knowledge will do. This same insight is the economic content of Carlsson and van Damme’s result.

Two rejoinders to the foregoing arguments why the risk-dominant equilibrium would be played are available. One rejoinder, specific to Carlsson and van Damme’s argument, is that the technical assumptions of their theorem are more restrictive than their account of the assumptions’ intuitive content would suggest, and that those technical assumptions are unlikely to hold exactly. This rejoinder can be rebutted, in turn, by the observation that assumptions of scientific theories invariably fail to correspond precisely to the truth. In a specific form, the rejoinder simply expresses an anti-scientific attitude, and it is not worthy of being taken seriously. At this point, there is nothing further

\(^{38}\)Actually, Carlsson and van Damme prove an even stronger conclusion than this.
to be said on either side. Probably neither party to the discussion will have persuaded the other, and their views will be as far apart as when they began. Just as it is not likely to produce scientific agreement, this line of argument is also unlikely to produce consensus about practical questions of competition policy or jurisprudence.

A much more compelling and interesting rejoinder is based on an argument made by Aumann (1990). To begin to set forth this rejoinder, let’s step back for a moment and put Carlsson and van Damme’s argument in context. Someone who uses that argument to argue that monopolistic outcomes cannot arise from tacit coordination intends, presumably, to contrast tacit coordination with weak explicit collusion involving overt negotiation of a cartel arrangement, and to assert that the cartel members could possibly reach common knowledge of their collusive intent through communication. But Aumann claims to refute that assertion.\(^{39}\) According to Aumann, a player can credibly communicate to an opponent that he would like the opponent to play the payoff-dominant strategy, but he cannot credibly communicate to the opponent that he will play the payoff-dominant strategy himself. The upshot is that the risk-dominant (that is, non-monopolistic) equilibrium is the only one that could rationally be played, even if there is unlimited opportunity for explicit communication among prospective colluders.

The conjunction of Aumann’s conclusion (that explicit collusion cannot be more feasible than tacit coordination is) and Carlsson and van Damme’s conclusion (that tacit coordination is not feasible) flies in the face of the incontrovertable evidence that explicit collusion has sometimes occurred and been stable through long intervals of time.\(^{40}\) Thus, if the soundness of Aumann’s argument is conceded, then Carlsson and van Damme’s conclusion must be denied.

Again, though, there is a rebuttal to the rejoinder. The rebuttal is that there is experimental evidence against Aumann’s view. Specifically, Charness

\(^{39}\)Moreover Harsanyi, a game theorist of unparalleled subtlety and insight, was persuaded by Aumann’s argument.

\(^{40}\)On the prevalence of cartels, see Levenstein and Suslow (2013).
(2000) reports an experiment in which a high proportion of subjects played a payoff-dominant, risk-dominated equilibrium after having had opportunity for explicit communication, but in which that equilibrium was rarely played by subjects who had not had such an opportunity. On the basis of this evidence, someone might decide to reject Aumann’s conclusion (even if he could not pinpoint where the logic of Aumann’s argument for it goes wrong), and could consequently hold the view that explicit collusion is likely to occur but that tacit coordination is implausible.

4.5 Initial capital investment as communication

So far in this section, we have supposed that industry conduct begins immediately with decisions about output or pricing, with capital investment implicitly having been determined previously and not being subject to adjustment. That is, investment (or capacity) is not a strategic choice. The argument of Aumann that has just been discussed, in particular, is formulated in that context.

Now, let’s reconsider that argument in the context of an industry with a two-stage life. In the first stage, an incumbent with large production capacity is joined by an entrant who bears an investment cost of acquiring capacity. (Whatever cost the incumbent may have borne in the past to build capacity, is now “sunk cost” and will not figure in the analysis that we make under these assumptions.) In the second stage, the firms will be quantity-setting competitors who choose their respective outputs a every date \((0, 1, 2, \ldots)\) subject to their respective capacity constraints. These constraints are at the exogenous (but non-binding) level for the incumbent and the level determined by stage-1 investment for the entrant, and cannot be adjusted during stage 2. At each date, firms produce perfect-substitute goods and the market price is perturbed by a demand shock that the firms cannot observe, as in the model of Green and Porter (1984).

As in the preceding discussion we simplify the analysis by supposing that

\(^{41}\text{Charness cites some previous research that also suggests that conclusion. See also Choi and Gerlach (2013).}\)
only finitely many—in this case, three—output levels are feasible. One is the static Cournot level. Call this output \( q_2 \). The other two feasible levels are parameters of the joint-profit-maximizing equilibrium of the Green-Porter environment that Abreu, Pearce, and Stacchetti (1986) (APS) have characterized. In that equilibrium, each firm produces output \( q_1 < q_2 \), which is approximately the static monopoly output, in the “cooperative” phase of equilibrium; and each produces a “maximin” output \( q_3 > q_2 \) during reversionary episodes. Thus we specify \( \{q_1, q_2, q_3\} \) to be the set of feasible output levels.

Given this assumption about feasible output levels, it is natural to specify also that, in stage 1, the entrant must choose a capacity level in \( \{q_1, q_2, q_3\} \).

Now, the question is, can the entrant credibly signal collusive intent and also motivate the incumbent to collude by choosing some particular level of capacity.

Of course, the entrant could do more than signalling—he could commit always to produce the collusive output—by choosing capacity \( q_1 \). But, if the entrant did that, then he would lose the capability to punish the incumbent for not reciprocating in collusion. The incumbent would make strictly higher profit in any period by producing \( q_2 \) than by producing \( q_1 \), and so would never produce \( q_1 \). The resulting industry conduct would not be collusive.

Nevertheless, the entrant would succeed in both signalling collusive intent and also motivating the incumbent to collude as well, if he were to choose capacity \( q_3 \). The rational incumbent should understand the implication of that choice in terms of a thought process that game theorists call forward induction. He should think as follows: If the entrant conjectured that I were going to play static Cournot, then he would be certain that he would never benefit (either directly or indirectly, by setting incentives for me) by producing more than \( q_2 \). Therefore, he would be sacrificing profit by making more costly investment to acquire capacity \( q_3 \). Since he is a profit maximizer, I deduce (by modus tollens) that his conjecture is that we are going to play the APS equilibrium rather than the static Cournot equilibrium in stage 2. Consequently, if I were to produce more than \( q_1 \) at the initial production date, then with high probability, the market price would be lower than the trigger price, and he—with certainty...
that I am behaving identically—would switch to the reversionary output level $q_3$, which would be bad for my profitability. That is, my conjecture is that he is going to produce the monopoly level of output as long as the market price stays above the trigger price, and my Nash response to that conduct is to do likewise.

The foregoing analysis follows closely the analysis of forward induction first proposed by van Damme (1989) for equilibrium of the *burning-the-dollar game*. That is a two-stage game, in which some particular single-stage game with several Nash equilibria is to be played in stage 2. In stage 1, one of the players has the opportunity publicly to burn a dollar bill. This action has no implication whatsoever for the feasibility of strategies or for the payoffs to any player of strategy profiles in stage 2. Nevertheless, the two-stage game has a unique equilibrium consistent with forward induction: the player does not burn the dollar, and even though he abstains from that action, the Nash equilibrium most favorable to him is played in stage 2. The analogy between the oligopoly game discussed here and the burning-the-dollar game is not exact, because the analogue of burning a dollar bill would be to invest in even more capacity than is required to play the APS equilibrium, and we have specified that option not to be available. Nevertheless, the logic of the incumbent’s thought process described above is precisely that spelled out in the analysis of burning the dollar.

Alternately, consider an industry such as production of a good that has just been invented by a non-producer (e.g., by a government laboratory), so that all prospective producers make simultaneous capacity choices in stage 1. Does forward induction also select the APS equilibrium in this environment? The answer is negative. Ben-Porath and Dekel (1992) have shown that, if *both* players of a single-stage game have the opportunity to publicly burn a dollar beforehand, then all equilibria of the single-stage game can be consistent with forward induction.

The upshot is that there is a theoretical model in which capacity choice functions as communication of collusive intent in the context of one, specific, initial configuration of an industry, but that the model does not show invest-
ment invariably to succeed to function in that way. In the current state of its development, game theory does not provide any more concrete guidance about how general may be the circumstances in which tacit coordination might arise via forward-induction reasoning.

4.6 Blundering into tacit coordination

In the introductory section of this chapter, we stated that, “There has been broad agreement in principle that monopoly conduct can arise spontaneously in highly concentrated markets that satisfy some other (possibly restrictive) conditions.” Building largely on the analysis provided by Aumann (1990) and Aumann and Brandenburger (1995) of Nash equilibrium as an outcome of rational interaction among players, we have found on the whole that the conditions for monopoly conduct to arise spontaneously as a result of such rational interaction are indeed restrictive.

But Aumann and Brandenburger provided sufficient conditions for players’ conjectures to be a Nash equilibrium, not necessary ones. They mentioned the possibility that players might “blunder into an equilibrium by accident,” even in a situation where their sufficient conditions for Nash equilibrium were not satisfied.

To assess this possibility requires a complement to the theory of outcomes of rational interaction under strong assumptions about higher-order knowledge of other players’ rationality: a theory of equilibrium as a long-run outcome of “blundering” by (predominantly) rational agents. Those agents can be modeled as making “optimal” decisions based on understanding their environment and on knowing what actions other players are using (on average, at least), but without consideration of what other agents might be thinking and of how their future actions might change as a result of their own optimization/reasoning processes.

This complementary theory is the topic of *evolutionary game theory*. Like other theories, this one is actually a family of related models. Virtually all evolutionary models have the implication that behavior almost surely approaches
some Nash equilibrium in the long run. The models that make the most precise predictions introduce a low level (made asymptotic to zero, in formal analysis) of irrational behavior that corresponds to mutation in a biological population. The canonical models of this sort were formulated by Young (1993) and by Kandori, Mailath, and Rob (1993). Analysis of those models provides a formal foundation the informal Aumann-Harsanyi argument that when one Nash equilibrium is payoff dominant and another one is risk dominant, play will converge to the risk-dominant equilibrium. As they relate to our analysis in Section 4.4, those models suggest that firms will not blunder into tacit coordination.

However, by relaxing only slightly the assumptions of those models about the optimality of non-mutant actions, Fudenberg and Imhof (2008) formulate a model in which the efficient outcome can be selected in repeated prisoner’s-dilemma games that are the abstract analogue of the repeated-interaction environments discussed earlier in this chapter. Thus, Fudenberg and Imhof’s model lends some support to a favorable view toward the possibility of competitors blundering into tacit coordination.

Sabourian and Juang (2008) ably survey general issues regarding use of evolutionary game theory to select the most plausible among the Nash equilibria in an environment. As they mention, the predictions of particular evolutionary models are sensitive to the specific assumptions that are incorporated in those models, and that might not be persuasive to a skeptic. Bergin and Lipman (1996) have shown that, if “mutations” are modeled as being state dependent, then any strict Nash equilibrium (that is, equilibrium in which each player’s Nash action is strictly preferred to all of his other actions) can be selected. The gist of Bergin and Lipman’s research is that, until economists develop intuitively compelling justifications for specific mutation processes, evolutionary game theory will remain an insightful but inconclusive framework for reasoning about whether “blundering” will lead to payoff-dominant (that is, tacitly coordinated) outcomes, or to risk-dominant (that is, non-collusively oligopolistic) ones.
5 Tacit coordination in antitrust litigation

A potentially collusive pattern of prices is a violation of the first section of the Sherman Act only if it is shown that there is an agreement among competitors. Unfortunately, there are no recipes to identify agreements from economic circumstantial evidence if the environment is such that collusive prices may arise without communication. Posner (2001) states that:

[I]n some circumstances competing sellers might be able to coordinate their pricing without conspiring in the usual sense of the term—that is, without any overt or detectable acts of communication. This is the phenomenon that lawyers call “conscious parallelism” and some economists term “oligopolistic interdependence,” but which I prefer to call “tacit collusion” in contrast to explicit collusion of the formal cartel or its underground counterpart. (Posner, 2001, pp.52–53)

In the same chapter, Posner (2001) tells the history of the price-fixing criminalization including the possibility of tacit coordination. He argues that just after the Sherman Act, judges and lawyers based their cases in the mere fact of explicit collusion and not in the economic consequences, probably because lawyers were more comfortable with conspiracy doctrine than with an economic theory of pricing. Posner (2001) points out the inadequacy of considering the existence of overt communication as the only decisive factor in collusion prosecution. Once the economic effects are taken into account, one must consider the possibility that a seemingly collusive price may have been reached without communication or even an agreement but from the understanding of the strategic interdependence. Posner (2001) approaches the issue of what constitutes a tacit agreement by referring to Turner (1962). For Turner (1962), whether a pattern of prices was reached through an agreement or not could be “considered purely as a problem in linguistic definition.” Posner (2001) then discusses when “plus factors” may allow the inference of explicit collusion from economic evidence.\footnote{See Kovacic et al. (2011) on identifying the strength of various plus factors.}
Baker (1993) also reached the conclusion that distinguishing an “agreement” among competitors when the evidence is “entirely circumstantial” is difficult from both a legal and an economic perspective:

Courts and commentators have debated for decades whether parallel price changes by oligopolists who recognize their interdependence provide a sufficient basis for a court to infer an unlawful horizontal agreement under Sherman Act 1, and if not what additional circumstantial evidence is required to prove a conspiracy. (Baker, 1993, p.144)

Baker (1993) concludes that regardless of the requirements to distinguish among coordination and agreement, antitrust policy should prevent practices that facilitate oligopoly coordination, which is the approach of the European Commission.

Turning to Europe, Garces-Tolon, Neven, and Seabright (2009) and Ivaldi et al. (2007) analyze the evolution of tacit coordination in Europe from 1990, year in which merger control became a responsibility of the European Union. Ivaldi et al. (2007) point out that tacit coordination, or collective dominance as it is called by the European Commission, has been a controversial issue. According to the article, the interpretation of collective dominance in litigation used to have no structural links and it was not until recent years. Consistency is up to some point due to a regulation in 2004 in which the Commission declared “incompatible with the EC treaties mergers that create or strengthen a dominant position as a result of which effective competition would be significantly impeded.” (Ivaldi et al., 2007, pp.217–18) In this sense, the European Commission is relying on game theory, mainly results on subgame perfect equilibrium in repeated games, to determine whether or not a concentration or a merger is strengthening a dominant position. An analysis of how results in repeated games have been used in litigation can be found in Garces-Tolon, Neven, and Seabright (2009). Ivaldi et al. (2007) also analyze the impact of game theory on European merger policy.
We hope this paper facilitates communication between lawyers and economists on these issues by providing a common language and framework for discussion. It is clear that antitrust enforcement authorities will wrestle with the identification of what types of conduct violate antitrust laws for years to come.
References


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